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Interfaces with Other Disciplines

Convex and nonconvex input-oriented technical and economic capacity measures: An empirical comparison[☆]

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ABSTRACT

This contribution has two main objectives. First, it aims to compare empirically input-oriented technical and economic capacity notions. Second, it aims to compare these capacity notions on both convex and nonconvex technologies. After defining these capacity notions, an empirical comparison is performed using a secondary data set containing data of French fruit producers. Anticipating two key empirical conclusions, we find that all these different capacity notions follow different distributions, and also that these distributions almost always differ under convex and nonconvex technologies.

1. Introduction

Analysing efficiency and productivity using frontier technologies has become a standard empirical tool serving a variety of academic, regulatory and managerial purposes. Indeed there is a huge academic literature applying these methodologies for analyzing private and public sector performance-related issues. Focusing on empirical surveys of certain well-studied sectors, one can point, for example, to banking (Harker & Zenios, 2001), education (Worthington, 2001), health care (Ozcan, 2008), insurance (Cummins & Weiss, 2000), justice system (Voigt, 2016) and real estate (Anderson, Lewis, & Springer, 2000). Apart from this surge of empirical applications, there has equally been an extended series of methodological innovations in this literature surveyed in, for example, Hatami-Marbini, Emrouznejad, and Tavana (2011) or Thanassoulis, Silva Portela, and Despić (2008).

An important area of regulatory applications has been the implementation of incentive regulatory mechanisms (e.g., price cap regulation) using frontier-based performance benchmarks in countries with liberalized network industries (e.g., electricity, gas, water utilities). One survey focusing on its use in the electricity sector is Jamasb and Pollitt (2000). An example of a managerial application is the use of frontier methods to save money by allowing use of internal funds to pursue a growth strategy in a US bank (see, e.g., Sherman & Ladino, 1995).

However, this frontier literature has largely ignored integrating the important notion of capacity utilization. Consequently, part of what appears like inefficiency may in fact be due to the short-run fixity of certain inputs, depending on the exact definition of capacity utilization. It is of equal importance to account for heterogeneity in capacity utilization when measuring productivity growth (e.g., Luh & Stefanou, 1991).

Capacity utilization of fixed inputs is relevant for both managers and policy makers at various levels of aggregation and in all economic sectors. For instance, at the country level capacity utilization is traditionally employed as a leading macro-economic indicator to forecast inflation (e.g., Christiano, 1981). The management of excess vessel capacities has recently become a key policy issue in fisheries due to degrading bio-stocks in this common pool resource. As an example, a variety of capacity measures has been employed to evaluate vessel decommissioning schemes (e.g., Walden, Kirkley, & Kitts, 2003). To curb overfishing, governments must determine sustainable capacity levels by implementing a variety of policy measures (e.g., licenses, fishing day restrictions, etc.). To define these policy measures, scientists have developed short-run industry models based on vessel capacity estimates to allow planning the industry and infer realistic decommissioning schemes (see, e.g., Lindebo, 2005).

However, different notions of capacity co-exist in the literature (e.g., Christiano, 1981 or Johansen, 1968). It is common to

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distinguish between technical or engineering concepts on the one hand and economic capacity concepts on the other hand. Johansen (1968) developed a technical or engineering approach by introducing a plant capacity notion. Plant capacity is defined as the maximal amount that can be produced per unit of time with existing plants and equipment without restrictions on the available variable inputs. This definition has been transposed into a production frontier context using output-oriented efficiency measures by Färe, Grosskopf, and Kokkelenberg (1989a).

Most economic capacity concepts are based on the cost function. In the literature there are basically at least three ways of defining a cost-based capacity notion (see, e.g., Nelson, 1989). Each of these notions attempts to isolate the short-run inadequate or excessive utilization of fixed inputs. A first notion of potential outputs is defined in terms of the outputs produced at short-run minimum average total cost given existing plant and input prices (for instance, Hickman, 1964). It stresses the need to exploit scale economies in the short-run. A second definition of potential outputs is conceived in terms of the outputs produced at minimum average total cost in the long-run (e.g., Cassels, 1937, among others). It is rarely used because its intertwining with the notion of scale economies. A third definition corresponds to the outputs at which the short-run and long-run average total cost curves are tangent. Since this tangency point is at the intersection of short-run and long-run expansion paths, this notion has considerable theoretical appeal (for example, Klein, 1960 or Segerson & Squires, 1990).

We are unaware of any study comparing this wide range of technical and economic capacity notions.[2] One plausible hypothesis explaining this lack of comparative studies is that the economic capacity notions at least implicitly adopt an input orientation, while the technical plant capacity notion is traditionally based on output-oriented efficiency measures. However, recently Cesaroni, Kerstens, and Van de Woestyne (2017) develop an input-oriented plant capacity notion based on input-oriented efficiency measures. Furthermore, Cesaroni, Kerstens, and Van de Woestyne (2019) recently defined new long-run output- and input-oriented plant capacity concepts. Therefore, a first major goal of this contribution is to make a theoretically coherent input-oriented comparison between this wide variety of technical and economic capacity notions. As a point of comparison, we also include the output-oriented plant capacity notion which has been used quite often in the literature in the last three decades since its inception (see Cesaroni et al., 2017 for a literature review).

This research assumes that different capacity concepts should ideally measure somehow a similar part of reality. Therefore, we require that these different capacity concepts satisfy some minimal consistency conditions in terms of both the comparability of distributions and the similarities in rankings.[3] Anticipating our empirical results, formal testing reveals that in almost all cases technical and economic capacity notions follow different distributions. These differences are confirmed in terms of rankings between input-oriented plant capacity and cost-based capacity notions under non-convexity, though less pronounced so under convexity.

It is well-known that the axiom of convexity has a potential impact on the empirical analysis based on technologies (see, e.g., Tone & Sahoo, 2003). In our context, for instance, Walden and Tomberlin (2010) document the effect of maintaining or dropping convexity on the output-oriented plant capacity utilization concept. Equally so, Cesaroni et al. (2017) reveal the impact of convexity on both the output- and input-oriented plant capacity utilization notions.

However, most researchers tend to ignore the potentially important impact of convexity on the cost function. This is related to a property of the cost function in the outputs that is ignored by most people. Indeed, some seminal contributions to axiomatic production theory indicate that the cost function is nondecreasing and convex in the outputs if and only if the technology is convex (e.g., Jacobsen, 1970). Otherwise, the cost function is nonconvex in the outputs. Briec, Kerstens, and Vanden Eeckaut (2004) refine this general property and prove that cost functions estimated on nonconvex technologies yield larger or equal cost estimates compared to cost functions estimated on convex technologies. Both these types of cost functions are identical when there is a single output and when constant returns to scale prevail. The large majority of empirical studies have failed to put these properties to a test. In our context, to the best of our knowledge the impact of convexity on cost-based notions of capacity utilisation has never been evaluated. Therefore, a second major goal of this contribution is to make a coherent input-oriented comparison between technical and economic capacity notions using both convex and nonconvex technologies to assess the impact of the convexity hypothesis. Again anticipating the empirical results, our formal tests show that almost all capacity concepts seem to follow a different distribution under convexity and nonconvexity, though convexity seems to matter less in terms of rankings.

This contribution is structured as follows. Section 2 summarizes the basic definitions of the technology and the cost function. The next Section 3 reviews in detail both the economic and technical capacity utilization definitions. This includes, among others, looking at the issue of normalization, given the existence of inefficiencies, and a priori determining the eventual impact of convexity. In the next Section 4 we develop an empirical illustration making use of an existing secondary data set, which makes our results replicable. The focus is on descriptive statistics, a formal testing of the resulting distributions, and a comparison of Spearman rank correlations. A final section concludes.

2. Technology and cost functions: basic definitions

In this section we define technology and some basic notation. Given N-dimensional input vectors $x \in \mathbb{R}^N_+$ and M-dimensional output vectors $y \in \mathbb{R}^M_+$, the production possibility set or technology T can be defined as $T = \{(x, y) \mid x \text{ can produce at least } y\}$. The input set $L(y) = \{x \mid (x, y) \in T\}$ associated with T holds all input vectors x capable of producing at least a given output vector y. In a similar way, the output set $P(x) = \{y \mid (x, y) \in T\}$ associated with T holds all output vectors y that can be produced from at most a given input vector x.

Throughout this contribution, technology T satisfies some combination of the following standard assumptions:

(T.1) Possibility of inaction and no free lunch, i.e., $(0, 0) \in T$ and if $(0, y) \in T$, then $y = 0$.
(T.2) T is a closed subset of $\mathbb{R}^N_+ \times \mathbb{R}^M_+$.
(T.3) Strong input and output disposal, i.e., if $(x, y) \in T$ and $(x', y') \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, then $(x', -y') \geq (x, -y) \Rightarrow (x', y') \in T$.
(T.4) $(x, y) \in T \Rightarrow \delta(x, y) \in T$ for $\delta \in \Gamma$, where:
 (i) $\Gamma \equiv \Gamma^{CRS} = \{\delta \mid \delta \geq 0\}$;
 (ii) $\Gamma \equiv \Gamma^{VRS} = \{\delta \mid \delta = 1\}$.
(T.5) T is convex.

[2] Sahoo and Tone (2009) come closest to comparing some technical and economic capacity notions in terms of inputs and costs using input-oriented nonparametric frontier models.

[3] This is inspired by the first two consistency conditions in the work of Bauer, Berger, Ferrier, and Humphrey (1998) regarding the evaluation of efficiency measures resulting from different frontier estimation methodologies. We are not convinced that the additional four consistency conditions make much sense in the framework of measuring and evaluating different capacity utilisation concepts.

Briefly discussing these traditional axioms on technology, it is useful to recall: (i) inaction is feasible, and there is no free lunch, (ii) closedness, (iii) free disposal of inputs and outputs, (iv) returns to scale assumptions (i.e., constant returns to scale (CRS) and variable returns to scale (VRS)), and (v) convexity of technology (see, e.g., Hackman, 2008 for details). Not all these axioms are maintained in the empirical analysis.[4] In particular, key assumptions distinguishing some of the technologies in the empirical analysis are CRS versus VRS, and convexity versus nonconvexity.

The input distance function completely characterizes the input set $L(y)$ and it can be defined as follows:

$$D_i(x, y \mid T) = \max\{\lambda \mid \lambda \geq 0, (x/\lambda, y) \in T\} \\ = \max\{\lambda \mid \lambda \geq 0, x/\lambda \in L(y)\}. \quad (1)$$

The main properties of this input distance function are: (i) $D_i(x, y|T) \geq 1$, with efficient production on the boundary (isoquant) of $L(y)$ represented by unity; (ii) it has a cost interpretation (see, e.g., Hackman, 2008).

The inverse of this input distance function $DF_i(x, y \mid T) = [D_i(x, y \mid T)]^{-1}$ is known as the radial input efficiency measure. Hence, the radial input efficiency measure is defined as:

$$DF_i(x, y \mid T) = \min\{\lambda \mid \lambda \geq 0, \lambda x \in L(y)\}. \quad (2)$$

Its key property is that it is situated between zero and unity $(0 < DF_i(x, y) \leq 1)$, with efficient production on the boundary (isoquant) of the input set $L(y)$ represented by unity.

Switching to a dual representation of technology, the cost function can be defined as the minimum expenditures needed to produce a given output vector y for a given vector of semi-positive input prices ($w \in \mathbb{R}^N_+$):

$$C(y, w \mid T) = \min_x\{wx \mid (x, y) \in T\} = \min_x\{wx : x \in L(y)\}. \quad (3)$$

Duality relations link these primal and dual representations of technology. Duality allows a well-behaved technology to be reconstructed from the observations on cost minimizing producer behavior, and the reverse. The duality between input distance function (1) and cost function (3) is:

$$D_i(x, y \mid T) = \min_w\{wx \mid C(y, w \mid T) \geq 1\}, x \in L(y), \quad (4)$$

$$C(y, w \mid T) = \min_x\{wx \mid D_i(x, y \mid T) \geq 1\}, w > 0. \quad (5)$$

It is common to establish such duality relations under the hypothesis of a convex technology or a convex input set (e.g., Hackman, 2008, Ch. 7). Briec et al. (2004) are the first to establish a local duality result between nonconvex technologies subject to various scaling laws and their corresponding nonconvex cost functions.

Next, the radial output efficiency measure can be defined as:

$$DF_o(x, y \mid T) = \max\{\theta \mid \theta \geq 0, \theta y \in P(x)\}, \quad (6)$$

and offers a complete characterization of the output set $P(x)$. Its main properties are that it is larger than or equal to unity $(DF_o(x, y|T) \geq 1)$, with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity, and that the radial output efficiency measure has a revenue interpretation (e.g., Hackman, 2008).

Partitioning the input vector into a fixed and variable part, we have $x = (x^f, x^v)$ with $x^f \in \mathbb{R}^{N_f}_+$ and $x^v \in \mathbb{R}^{N_v}_+$ such that $N = N_f + N_v$. Furthermore, we can make the same distinction regarding the input price vector $w = (w^f, w^v)$.

In a similar way to Färe, Grosskopf, and Valdmanis (1989b), a short-run technology $T^f = \{(x^f, y) \in \mathbb{R}^{N_f}_+ \times \mathbb{R}^M_+ \mid (x^f, x^v)$ can produce at least $y\}$ and the corresponding input set $L^f(y) = \{x^f \in \mathbb{R}^{N_f}_+ \mid (x^f, y) \in T^f\}$ and output set $P^f(x^f) = \{y \mid (x^f, y) \in T^f\}$ can be defined. Note that technology T^f does not include variable inputs, and the maximal output from this technology is determined solely by the fixed inputs. This yields an equivalent output level as a technology where all the variable inputs are set to zero. Thus, technology T^f is obtained by a projection of technology $T \subset \mathbb{R}^N_+ \times \mathbb{R}^M_+$ into the subspace $\mathbb{R}^{N_f}_+ \times \mathbb{R}^M_+$: i.e., by setting all variable inputs equal to zero. This projection maps (x^f, x^v, y) onto $(x^f, 0, y)$ which is mathematically identified with (x^f, y).[5] The same applies by analogy to the input set $L^f(y)$ and the output set $P^f(x^f)$.

By analogy, the short-run total cost function is defined as follows:

$$C(w, x^f, y \mid T) = \min_{x^v}\{w^v x^v + w^f x^f \mid (x^f, x^v, y) \in T\}. \quad (7)$$

The short-run variable cost function is defined as:

$$VC(w^v, x^f, y \mid T) = \min_{x^v}\{w^v x^v \mid (x^f, x^v, y) \in T\}. \quad (8)$$

Note that the short-run total cost function is simply the sum of the short-run variable cost function and the observed fixed costs.

The sub-vector input efficiency measure reducing only the variable inputs is defined as follows:

$$DF_i^{SR}(x^f, x^v, y \mid T) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(y)\}. \quad (9)$$

Next, we need the particular input set $L(0) = \{x \mid (x, 0) \in T\}$ for which the output level is set to at least zero. The input efficiency measure reducing all inputs relative to this input set with zero output level is given by:

$$DF_i(x, 0 \mid T) = \min\{\lambda \mid \lambda \geq 0, \lambda x \in L(0)\}. \quad (10)$$

Then, the sub-vector input efficiency measure reducing variable inputs evaluated relative to this input set with zero output level is given by:

$$DF_i^{SR}(x^f, x^v, 0 \mid T) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(0)\}. \quad (11)$$

By analogy, denote the radial output efficiency measure of the output set $P^f(x^f)$ by $DF_o^f(x^f, y)$. This efficiency measure can be defined as

$$DF_o^f(x^f, y \mid T) = \max\{\theta \mid \theta \geq 0, \theta y \in P^f(x^f)\}. \quad (12)$$

Next, we introduce the particular output set $P = \{y \mid \exists x : (x, y) \in T\}$ containing all possible outputs regardless of the required inputs. This set allows us to define a new efficiency measure $DF_o(y|T)$ that does not depend on a particular input vector x:

$$DF_o(y \mid T) = \max\{\theta \mid \theta \geq 0, \theta y \in P\}. \quad (13)$$

Contrary to the radial output efficiency measure (6), this new efficiency measure $DF_o(y|T)$ is allowed to choose the inputs needed for maximizing θ.

Now, for K observations $(x_k, y_k) \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, $(k = 1, \ldots, K)$ a unified algebraic representation of convex and nonconvex nonparametric frontier technologies under CRS and VRS assumptions is possible as follows:

$$T^{\Lambda,\Gamma} = \left\{(x, y) \mid x \geq \sum_{k=1}^K x_k \delta z_k, y \leq \sum_{k=1}^K y_k \delta z_k, z \in \Lambda, \delta \in \Gamma\right\}, \quad (14)$$

where

(i) $\Gamma \equiv \Gamma^{CRS} = \{\delta \mid \delta \geq 0\}$;

[4] Note that the convex VRS technology does not satisfy inaction.

[5] In order to see this projection operationalized, start from model (2) in Appendix B and set $x_k^v = 0$ for all k. Consequently, the variable input constraints become $0 \leq 0$ which is always satisfied and so can be removed without altering the outcome. The resulting model is model (3) of Appendix B.

(ii) $\Gamma \equiv \Gamma^{\text{VRS}} = \{\delta \mid \delta = 1\}$;

and

(i) $\Lambda \equiv \Lambda^{\text{C}} = \left\{ z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } \forall k \in \{1, \ldots, K\} : z_k \geq 0 \right\}$;

(ii) $\Lambda \equiv \Lambda^{\text{NC}} = \left\{ z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } \forall k \in \{1, \ldots, K\} : z_k \in \{0, 1\} \right\}$.

Observe there is one activity vector z operating subject to a nonconvexity or convexity constraint as well as a scaling parameter δ allowing for some particular scaling of all K observations determining the technology. The activity vector z having real valued components summing to unity represents the convexity axiom. This activity vector with binary valued components summing to unity corresponds with nonconvexity. The scaling parameter δ is free under CRS and fixed at the unit level under VRS.

To compute the input efficiency measure (2) or cost function (3) relative to convex technologies in (14) requires solving nonlinear programming (NLP) problems for each evaluated observation. These NLPs can be easily transposed into the familiar linear programming (LP) problems found in the literature (see Hackman, 2008).[6] For the nonconvex technologies, nonlinear binary mixed integer programs must be solved, but alternative solution strategies are available (see Kerstens & Van de Woestyne, 2014).

From here on, the above notions of efficiency measures and cost functions are conditioned relative to nonparametric VRS or CRS technologies satisfying either convexity (denoted C) or nonconvexity (denoted NC).

3. Economic and technical capacity utilization: literature review and definitions

A variety of capacity notions coexist in the economic literature. It is customary to distinguish between technical (engineering) and economic (mainly cost-based) capacity concepts (see, e.g., Johansen, 1968; Nelson, 1989). We first address the economic concepts using a cost function approach, and then turn to the technical or engineering notion.

3.1. Economic capacity concepts

At least three ways of defining a cost-based notion of capacity have been proposed in the literature (see Nelson, 1989). Each of these notions aims to isolate the short-run excessive or inadequate utilization of existing fixed inputs (e.g., capital stock). A first notion is defined in terms of the output produced at short-run minimum average total cost given existing input prices (see Hickman, 1964, among others). A second definition focuses on the outputs for which short-run and long-run average total costs curves are tangent (e.g., Segerson & Squires, 1990). This tangency point notion is known under two variations depending on what are supposed to be the decision variables. One notion assumes that outputs are constant and determines optimal variable and fixed inputs. Another notion assumes that fixed inputs cannot adjust, but outputs, output prices and fixed input prices do adjust. A third and final definition of economic capacity considers the output determined by the minimum of the long-run average total costs (e.g., Cassels, 1937; Klein, 1960).

To apply these notions of economic capacity utilization using nonparametric frontier technologies, one can characterize the above three economic capacity notions, one of which has two variants, in a multiple output context in the following series of definitions (see, e.g, De Borger, Kerstens, Prior, & Van de Woestyne, 2012).

Definition 3.1. The minimum of the short-run total cost function $C(y, w^v, x^f \mid VRS)$ is $C(y, w^v, x^f \mid CRS)$.

The minimum of the single output short-run average total cost function can be determined indirectly in the multiple output case by solving for a variable cost function relative to a CRS technology $(VC(y, w^v, x^f \mid CRS))$, and simply adding observed fixed costs $FC = w^f x^f$. The resulting short-run total cost function $C(y, w^v, x^f \mid CRS) = (VC(y, w^v, x^f \mid CRS) + FC)$ offers the reference point for this capacity notion. In the convex case, computing a cost function boils down to a well-known linear program. But, in the nonconvex case one must solve a mixed binary integer linear program.

Definition 3.2. Let x^{f*} represent optimal fixed inputs, $p \in \mathbb{R}^N_+$ a vector of input prices, and $y(p, w^f, x^f)$ the outputs that have been adjusted in terms of given output prices, fixed input prices and the given fixed inputs. Then,

(i) tangency cost with modified fixed inputs $C^{tang1}(y, w, x^{f*} \mid VRS)$ is

$$C(y, w \mid VRS) = C(y, w^v, x^{f*} \mid VRS);$$

(ii) tangency cost with modified outputs $C^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS)$ is

$$C(y(p, w^f, x^f), w \mid VRS) = C(y(p, w^f, x^f), w^v, x^f \mid VRS).$$

First, the tangency point between short- and long-run costs can also be estimated using nonparametric cost frontiers. Two tangency points can be derived depending on the choice of decision variables.

One tangency cost notion assumes that outputs remain constant and then determines optimal variable and fixed inputs $C^{tang1}(y, w, x^{f*} \mid VRS)$. This can be solved indirectly by minimizing a long-run total cost function $C(y, w \mid VRS)$ yielding optimal fixed inputs (x^{f*}). By definition, the short-run and total cost function with fixed inputs equal to these ex post optimal fixed inputs $FC(y, w^v, x^{f*} \mid VRS)$ yields exactly the same solution in terms of optimal costs and optimal variable inputs $C(y, w^v, x^{f*} \mid VRS) = VC(y, w^v, x^{f*} \mid VRS) + FC(y, w^v, x^{f*} \mid VRS)$. Hence, the optimal solution for $C(y, w \mid VRS)$ generates the tangency point we are looking for. In the convex case, computing this cost function requires solving again a linear program. In the nonconvex case, one needs to solve a mixed binary integer linear programming problem.

Another tangency point, favored by Nelson (1989, p. 277) and analyzed in detail in Briec, Kerstens, Prior, and Van de Woestyne (2010), assumes that fixed inputs cannot be adjusted in the short-run, but that outputs, output prices ($p \in \mathbb{R}^M_+$) and fixed input prices are adjustable such that installed capacity is utilized ex post at a tangency cost level ($C^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS)$). Though one may object that outputs are assumed to be exogenous in a competitive cost minimization model, this tangency notion offers a useful reference point, since it retrospectively indicates the output quantities and prices as well as the fixed input prices at which existing fixed inputs would have been optimally utilized. For an arbitrary observation, this tangency cost level may imply an output level ($y(p, w^f, x^f)$) below or above current outputs. In the convex case, optimal costs at this tangency point are determined by solving for each observation a nonlinear system of inequalities (Briec et al., 2010). In the nonconvex case, however, one must solve for each observation a mixed binary integer nonlinear system of inequalities.

Definition 3.3. The minimum of the long-run total cost function $C(y, w \mid VRS)$ is obtained as $C(y, w \mid CRS)$.

[6] By substituting $t_k = \delta z_k$ in (14), one can rewrite the sum constraint on the activity vector z. Note that the constraints on the scaling factor are integrated into the latter sum constraint and the LP appears.

The minimum of long-run average total costs can be determined indirectly by solving for a long-run total cost function defined relative to a *CRS* technology $C(y, w \mid CRS)$. In the convex case, computing this cost function again involves solving a linear program. For the nonconvex case, one must solve a mixed binary integer linear programming problem. For convenience, the way of computing all economic capacity concepts in the convex as well as nonconvex case are spelled out in the Appendix B.

In a frontier context, some of the above cost-based capacity concepts or some combination there-off have been reported in Giménez and Prior (2007), Prior-Jiménez (2003), or Sahoo and Tone (2009), among others. Note that we have ignored the discussion of alternative capacity concepts based on the revenue function (e.g., Lindebo, Hoff, & Vestergaard, 2007) or the profit function (e.g., Coelli, Grifell-Tatjé, & Perelman, 2002).

3.2. Plant capacity concepts

Johansen (1968) proposed a plant capacity notion that has been made operational by Färe et al. (1989a) and Färe et al. (1989b) using a pair of output-oriented efficiency measures. The plant capacity notion is defined by Johansen as "the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." Cesaroni et al. (2017) develop a plant capacity notion using a pair of input-oriented efficiency measures. All of these proposals use *VRS* technologies. We now recall these definitions of the output- and input-oriented plant capacity utilization.

Definition 3.4. The short-run output-oriented plant capacity utilization (PCU_o^{SR}) is defined as:

$$PCU_o^{SR}(x, x^f, y \mid VRS) = \frac{DF_o(x, y \mid VRS)}{DF_o^f(x^f, y \mid VRS)}, \tag{15}$$

where $DF_o(x,y|VRS)$ and $DF_o^f(x^f, y \mid VRS)$ are output efficiency measures relative to *VRS* technologies including respectively excluding the variable inputs as defined before. Notice that $0 < PCU_o^{SR}(x, x^f, y \mid VRS) \leq 1$, since $1 \leq DF_o(x, y \mid VRS) \leq DF_o^f(x^f, y \mid VRS)$. Thus, output-oriented plant capacity utilization has an upper limit of unity, but no lower limit. This output-oriented plant capacity utilisation compares the maximum amount of outputs with given inputs to the maximum amount of outputs in the sample with potentially unlimited amounts of variable inputs: hence it is smaller than unity. It answers the question how the current amount of efficient outputs relates to the maximal possible amounts of efficient outputs given unlimited amounts of variable inputs. Following the terminology introduced by Färe et al. (1989a), Färe et al. (1989b) and Färe, Grosskopf, and Lovell (1994) one can distinguish between a so-called biased plant capacity measure $DF_o^f(x^f, y \mid VRS)$ and an unbiased plant capacity utilization measure $PCU_o^{SR}(x, x^f, y \mid VRS)$, where the ratio of efficiency measures ensures to eliminate any existing inefficiency.

Cesaroni et al. (2017) define a new input-oriented plant capacity measure as follows:

Definition 3.5. The short-run input-oriented plant capacity utilization (PCU_i^{SR}) is defined as:

$$PCU_i^{SR}(x, x^f, y \mid VRS) = \frac{DF_i^{SR}(x^f, x^v, y \mid VRS)}{DF_i^{SR}(x^f, x^v, 0 \mid VRS)}, \tag{16}$$

where $DF_i^{SR}(x^f, x^v, y \mid VRS)$ and $DF_i^{SR}(x^f, x^v, 0 \mid VRS)$ are both sub-vector input efficiency measures reducing only the variable inputs relative to the technology, whereby the latter efficiency measure is evaluated at a zero output level.[7] Notice that $PCU_i^{SR}(x, x^f, y \mid VRS) \geq 1$, since $0 < DF_i^{SR}(x^f, x^v, 0 \mid VRS) \leq DF_i^{SR}(x^f, x^v, y \mid VRS)$. Thus, input-oriented plant capacity utilization has a lower limit of unity, but no upper limit. This input-oriented plant capacity utilisation compares the minimum amount of variable inputs for given amounts of outputs with the minimum amount of variable inputs with output levels where production is initiated: hence it is larger than unity. It answers the question how the amount of variable inputs compatible with the initialisation of production must be scaled up to produce the current amount of outputs. Similar to the previous case, one can distinguish between a so-called biased plant capacity measure $DF_i^{SR}(x^f, x^v, 0 \mid VRS)$ and an unbiased plant capacity utilization measure $PCU_i^{SR}(x, x^f, y \mid VRS)$, the latter being cleaned of any prevailing inefficiency.

Cesaroni et al. (2019) define new long-run output- and input-oriented plant capacity concepts.

Definition 3.6. The long-run output-oriented plant capacity utilization (PCU_o^{LR}) is defined as:

$$PCU_o^{LR}(x, y \mid VRS) = \frac{DF_o(x, y \mid VRS)}{DF_o(y \mid VRS)}, \tag{17}$$

where $DF_o(x, y|VRS)$ and $DF_o(y|VRS)$ are output efficiency measures relative to technologies including all inputs respectively ignoring all inputs. Notice that $0 < PCU_o^{LR}(x, y \mid VRS) \leq 1$, since $1 \leq DF_o(x, y|VRS) \leq DF_o(y|VRS)$. Thus, long-run output-oriented plant capacity utilisation has an upper limit of unity, but no lower limit. This long-run output-oriented plant capacity utilisation compares the maximum amount of outputs with given inputs to the maximum amount of outputs in the sample with potentially unlimited amounts of both fixed and variable inputs: hence it is smaller than unity. It answers the question how the current amount of efficient outputs relates to the maximal possible amounts of efficient outputs given unlimited amounts of inputs. Again, it is possible to distinguish between a so-called biased plant capacity measure $DF_o(y|VRS)$ and an unbiased plant capacity utilization measure $PCU_o^{LR}(x, y \mid VRS)$ that is free of any inefficiency.

Definition 3.7. The long-run input-oriented plant capacity utilization (PCU_i^{SR}) is defined as:

$$PCU_i^{LR}(x, y \mid VRS) = \frac{DF_i(x, y \mid VRS)}{DF_i(x, 0 \mid VRS)}, \tag{18}$$

where $DF_i(x, y|VRS)$ and $DF_i(x, 0|VRS)$ are both input efficiency measures aimed at reducing all input dimensions relative to the technology, whereby the latter efficiency measure is evaluated at a zero output level. Notice that $PCU_i^{LR}(x, y \mid VRS) \geq 1$, since $0 < DF_i(x, 0|VRS) \leq DF_i(x, 0|VRS) \leq 1$.[8] Thus, long-run input-oriented plant capacity utilisation has a lower limit of unity, but no upper limit. This long-run input-oriented plant capacity utilisation compares the minimum amount of all inputs for given amounts of outputs with the minimum amount of all inputs with outputs where production is initiated: hence it is larger than unity. It answers the question how the amount of all inputs compatible with the initialisation of production must be scaled up to produce the current amount of outputs. Once more, one can distinguish between a so-called biased plant capacity measure $DF_i(x, 0|VRS)$ and an unbiased

[7] An equivalent formulation for $DF_i^{SR}(x^f, x^v, 0 \mid VRS)$ is $DF_i^{SR}(x^f, x^v, y_{min} \mid VRS)$, where $y_{min} = \min_{k=1,\dots,K} y_k$ whereby the minimum is taken in a component-wise manner for every output over all observations: see Proposition B.1 in Appendix B.

[8] An equivalent formulation for $DF_i(x, 0|VRS)$ is $DF_i(x, y_{min}|VRS)$, where $y_{min} = \min_{k=1,\dots,K} y_k$ whereby the minimum is taken in a component-wise manner for every output over all observations: see Proposition B.2 in Appendix B.

plant capacity utilization measure $PCU_i^{LR}(x, y \mid VRS)$ that is unaffected by any inefficiency. Given the recent date of the introduction of both the short-run input-oriented plant capacity measure on the one hand, and the long-run plant capacity notions on the other hand, we provide some more background information in the Appendix A.

We note that the majority of capacity concepts presumes the existence of fixed inputs distinct from variable inputs. But, both the economic capacity concept of the minimum of the long-run total cost function (i.e., Definition 3.3) and the long-run plant capacity concepts (i.e., Definitions 3.6 and 3.7) dissent from this view and assume that all inputs are subject to change.

While these definitions in itself are sufficiently clear, it may be useful to underscore that both these short-run concepts differ with respect to the property of attainability. As stressed by Johansen (1968, p. 362), the extra variable inputs necessary to reach the maximal plant capacity output may not be available at the firm level (or the change in variable inputs is not necessarily costless), rendering the short-run output-oriented plant capacity notion unattainable. Furthermore, even if these extra variable inputs are available at the firm level, restrictions on the available extra variable inputs at the sector level may prevent all firms from simultaneously reaching their maximal capacity output (or the change in variable inputs may imply substantial costs). By contrast, the short-run input-oriented plant capacity notion is always attainable in that one can always reduce the amount of existing variable inputs such that one reaches an input set with zero output level. Doing so is possible at the firm level as well as at the sectoral level. The same reasoning applies to the corresponding long-run plant capacity concepts. The reader is referred to the work of Kerstens, Sadeghi, and Van de Woestyne (2019) for further discussion on the issue of attainability.

In the convex case, computing these plant capacity measures involve solving a linear program for each observation. For the nonconvex case, one must solve a mixed binary integer linear programming problem. For convenience, the way of computing all plant capacity concepts in the convex as well as nonconvex case are spelled out in the Appendix B.

3.3. Economic capacity concepts: normalization and impact of convexity

Since the literature has abundantly shown that inefficiencies are part and parcel of economic life, following the plant capacity concepts it may be useful to normalize the economic capacity concepts as well. We are inspired by the notion of overall efficiency (see Färe et al., 1994 or Hackman, 2008), whereby in the case of the cost function one divides the minimal cost by the observed costs (wx). Starting from the Definitions 3.1, 3.2 and 3.3, we can now define the normalized economic capacity utilization concepts as follows:

Definition 3.8.

(i) The normalized minimum of the short-run total cost function $C_N(y, w^v, x^f \mid VRS)$ is $C(y, w^v, x^f \mid CRS)/wx$.
(ii) The normalized tangency cost with modified fixed inputs $C_N^{tang1}(y, w, x^{f*} \mid VRS)$ is $C(y, w \mid V)/wx = C(y, w^v, x^{f*} \mid VRS)/wx$.
(iii) The normalized tangency cost with modified outputs $C_N^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS)$ is $C(y(p, w^f, x^f), w \mid VRS)/wx = C(y(p, w^f, x^f), w^v, x^f \mid VRS)/wx$.
(iv) The normalized minimum of long-run total cost function is defined as $C_N(y, w \mid VRS)$ is $C(y, w \mid CRS)/wx$.

Notice that all of these normalized economic capacity utilization concepts are bounded above at unity, except for the normalized tangency cost with modified outputs $C_N^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS)$ which can be smaller or larger than unity. To understand this phenomenon we must first realize that for observed outputs, we have: $C(y, w \mid VRS) \neq C(y, w^v, x^f \mid VRS)$. As a consequence, in Definition 3.2 the optimal tangency cost may be smaller or larger to each of the sides of this inequality. To be explicit, on the one hand we obtain $C(y(p, w^f, x^f), w \mid VRS) = C(y(p, w^f, x^f), w^v, x^f \mid VRS) \gtreqless C(y, w \mid VRS)$, and on the other hand we get: $C(y(p, w^f, x^f), w \mid VRS) = C(y(p, w^f, x^f), w^v, x^f \mid VRS) \gtreqless C(y, w^v, x^f \mid VRS)$.

Finally, when comparing convex and nonconvex results, there are cases where plant and economic capacity concepts can be ordered a priori. First, we state these results for the biased plant capacity concepts as well as the non-normalized economic capacity concepts.

Proposition 3.1.

(i) *For the output-oriented plant capacity, we have:* $DF_o^f(x^f, y \mid VRS, C) \geq DF_o^f(x^f, y \mid VRS, NC)$.
(ii) *For the input-oriented plant capacity, we have:* $DF_i^{SR}(x^f, x^v, 0 \mid VRS, C) \leq DF_i^{SR}(x^f, x^v, 0 \mid VRS, NC)$.
(iii) *For the output-oriented plant capacity, we have:* $DF_o(y|VRS, C) \geq DF_o(y|VRS, NC)$.
(iv) *For the input-oriented plant capacity, we have:* $DF_i(x, 0|VRS, C) \leq DF_i(x, 0|VRS, NC)$.
(v) *For the minimum of the short-run total cost function, we have:* $C(y, w^v, x^f \mid VRS, C) \leq C(y, w^v, x^f \mid VRS, NC)$.
(vi) *For the tangency cost with modified fixed inputs, we have:* $C^{tang1}(y, w, x^{f*} \mid VRS, C) \leq C^{tang1}(y, w, x^{f*} \mid VRS, NC)$.
(vii) *For the tangency cost with modified outputs, we have:* $C^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS, C) \gtreqless C^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS, NC)$.
(viii) *For the minimum of long-run total cost function, we have:* $C(y, w \mid VRS, C) \leq C(y, w \mid VRS, NC)$.

The proof of this Proposition 3.1 is in Appendix C. Thereafter, we do the same for unbiased plant capacity utilization concepts and the normalized economic capacity utilization concepts.

Proposition 3.2.

(i) *For the short-run output-oriented plant capacity utilization, we have:* $PCU_o^{SR}(x, x^f, y \mid VRS, C) \gtreqless PCU_o^{SR}(x, x^f, y \mid VRS, NC)$.
(ii) *For the short-run input-oriented plant capacity utilization, we have:* $PCU_i^{SR}(x, x^f, y \mid VRS, C) \gtreqless PCU_i^{SR}(x, x^f, y \mid VRS, NC)$.
(iii) *For the long-run output-oriented plant capacity utilization, we have:* $PCU_o^{LR}(x, x^f, y \mid VRS, C) \gtreqless PCU_o^{LR}(x, x^f, y \mid VRS, NC)$.
(iv) *For the long-run input-oriented plant capacity utilization, we have:* $PCU_i^{LR}(x, x^f, y \mid VRS, C) \gtreqless PCU_i^{LR}(x, x^f, y \mid VRS, NC)$.
(v) *For the minimum of the short-run total cost function, we have:* $C_N(y, w^v, x^f \mid VRS, C) \leq C_N(y, w^v, x^f \mid VRS, NC)$.
(vi) *For the tangency cost with modified fixed inputs, we have:* $C_N^{tang1}(y, w, x^{f*} \mid VRS, C) \leq C_N^{tang1}(y, w, x^{f*} \mid VRS, NC)$.
(vii) *For the tangency cost with modified outputs, we have:* $C_N^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS, C) \gtreqless C_N^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS, NC)$.
(viii) *For the minimum of long-run total cost function, we have:* $C_N(y, w \mid VRS, C) \leq C_N(y, w \mid VRS, NC)$.

The proof of this Proposition 3.2 is in Appendix C.

Both Propositions 3.1 and 3.2 form the basis for our statistical test comparing different capacity notions among themselves and in relation to the convexity axiom. We opt for a formal test statistic proposed by Li (1996) and refined by Fan and Ullah (1999) and Li, Maasoumi, and Racine (2009) lately (henceforth Li-test). The null hypothesis of this Li-test states that both distributions are equal

Table 1
Descriptive statistics for French fruit producers (1984-1986).

Variable	Trimmed mean[a]	Minimum	Maximum
Capital (fixed input)	85,602.58	8891	500,452
Labor (variable input	229,569	79569	1,682,201
Materials (variable input)	157,610.9	19566	1,523,776
Volume of apple production (output)	2.146273	0.00061	37.98153
Volume of other products (output)	1.37793	0.000672	25.895
Price of capital	1.167934	0.167802	7.889478
Price of labor	1.059968	0.492821	1.771435
Price of materials	6.72676	1.732421	22.61063

Note: [a] 10% trimming level.

Table 2
Descriptive statistics for all biased and non-normalized capacity notions.

Convex	BPC_o^{LR}	BPC_i^{LR}	BPC_o^{SR}	BPC_i^{SR}	SRC	C^{tang1}	C^{tang2}	LRC
Average	18.33059	0.430065	5.414862	0.42333	620247.8	718839.9	315274.8	511506.1
Stand. Dev.	22.62704	0.201673	4.678063	0.194978	827159.9	1124454	1058872	758764.8
Minimum	1	0.047301	1	0.047301	10454.19	150112.7	132380.2	8507.063
Maximum	190.4508	1	35.29532	1	6238552	11815722	21170527	6095270
Nonconvex	BPC_o^{LR}	BPC_i^{LR}	BPC_o^{SR}	BPC_i^{SR}	SRC	C^{tang1}	C^{tang2}	LRC
Average	7.639567	0.435916	2.891018	0.430783	816915.6	1160906	301561.7	683063.1
Stand. Dev.	9.863395	0.206526	2.935252	0.202152	981389.5	1730077	1043655	880893.2
Minimum	1	0.047301	1	0.047301	14486.9	150112.7	132380.2	13147.43
Maximum	96.51148	1	32.45654	1	7100639	13448388	21170527	6754195

BPC_o^{LR}: Biased long-run output-oriented plant capacity ($DF_o(y|VRS, .)$).
BPC_i^{LR}: Biased long-run input-oriented plant capacity ($DF_i(x, 0|VRS, .)$).
BPC_o^{SR}: Biased short-run output-oriented plant capacity ($DF_o^f(x^f, y \mid VRS, .)$).
BPC_i^{SR}: Biased short-run input-oriented plant capacity ($DF_i^{SR}(x^f, x^v, 0 \mid VRS, .)$).
SRC: Non-normalized short-run total cost ($C(y, w^v, x^f \mid CRS, .)$).
C^{tang1}: Non-normalized tangency cost with modified fixed inputs ($C^{tang1}(y, w, x^{f*} \mid VRS, .)$).
C^{tang2}: Non-normalized tangency cost with modified outputs ($C^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS, .)$).
LRC: Non-normalized long-run total cost($C(y, w \mid CRS, .)$).

for a given efficiency score or cost frontier estimate and for a given underlying specification of technology. The alternative hypothesis is simply that both distributions are different. This test is valid for both dependent and independent variables. Note that dependency is a characteristic of frontier estimators: frontier efficiency and cost levels depend on sample size, among others. Now, we are in a position to start developing the empirical illustration.

4. Empirical illustration

4.1. Data

To illustrate how the economic and plant capacity notions can be used, we draw upon a secondary data set that is publicly available from the Journal of Applied Econometrics Data Archive.[9] This guarantees the replicability of all our empirical results. We opt for an unbalanced panel of three years (1984-1986) of French fruit producers based on annual accounting data collected in a survey (see Ivaldi, Ladoux, Ossard, & Simioni, 1996 for details). Two main criteria determined the selection of farms: (i) the production of apples must be larger than zero, and (ii) the productive acreage of the orchard must be at least five acres. Three aggregate inputs are combined to produce two outputs. The three inputs are: (i) capital (including land), (ii) labor, and (iii) materials. The two aggregate outputs are (i) the production of apples, and (ii) an aggregate of alternative products. Also input prices are available in French francs. The first input capital is considered as fixed.

Summary statistics for the 405 observations in total and details on the definitions of all variables are available in Appendix 2 in Ivaldi et al. (1996). Observe that the limited length of the panel (just three years) justifies the use of an intertemporal frontier accumulating all observations in the technology: this approach fundamentally ignores technical change.

Table 1 presents basic descriptive statistics for the inputs, the outputs, and the input prices. One observes basically a lot of heterogeneity and a rather wide range for all inputs and outputs. The range for some of the input prices is smaller. More details on the data are available in Ivaldi et al. (1996).

In the following, we first discuss the biased plant capacity and non-normalized economic capacity notions. Thereafter, we study the unbiased plant capacity utilization and normalized economic capacity utilization (CU) notions.

4.2. Comparing biased and non-normalized capacity notions

Table 2 shows basic descriptive statistics for all biased and non-normalized capacity notions. We report the average, the standard deviation, and the minima and maxima depending on the context. The relations between convex and nonconvex results are conditioned by the relations described in Proposition 3.1. First, ignoring the capacity notion that cannot be ranked (i.e., C^{tang2}), on average convex and nonconvex results are rather markedly different, except for BPC_i^{SR} and BPC_i^{LR} where the difference is quite small. Second, the range of the results are sometimes different, but some share one of the extremes, except for BPC_i^{SR}, BPC_i^{LR}, and C^{tang2} for which the range is identical.

Table 3 reports the Li-test results and is structured as follows. First, components on the diagonal (in bold) depict the Li-test statistic between the convex and nonconvex cases. Second, the components under the diagonal show the Li-test statistic between convex capacities, and the components above the diagonal show the Li-test statistic between nonconvex capacities. The following

[9] Web site: http://qed.econ.queensu.ca/jae.

Table 3
Li-test between all biased and non-normalized capacity notions.

Variables	BPC_o^{LR}	BPC_i^{LR}	BPC_o^{SR}	BPC_i^{SR}	SRC	C^{tang1}	C^{tang2}	LRC
BPC_o^{LR}	**22.6832*** **	*174.6714*** *	*26.6822*** *	*175.7298*** *	*288.4541*** *	*289.8256*** *	*296.0598*** *	*118.6018*** *
BPC_i^{LR}	248.2363***	**−1.5052***	113.4045***	−1.5551*	173.2542***	173.1617***	296.0599***	173.2206***
BPC_o^{SR}	35.9509***	172.5672***	**24.7981*** **	*115.1117*** *	*288.4545*** *	*289.8256*** *	*296.0598*** *	*288.4553*** *
BPC_i^{SR}	175.6912***	−1.5634*	173.8835***	**−1.4369***	*288.4547*** *	*176.1023*** *	*296.0599*** *	*176.0521*** *
SRC	288.6066***	173.2392***	288.6082***	174.2108***	**3.7014*** **	*16.5229*** *	*79.0476*** *	*0.4898*
C^{tang1}	290.8352***	290.8353***	133.9842***	174.24***	32.1426***	**10.798*** **	*104.1275*** *	*27.8555*** *
C^{tang2}	295.933***	295.9333***	295.9331***	174.2678***	66.8757***	67.0158***	**−2.4851*** **	*80.1807*** *
LRC	288.5402***	288.5424***	288.5418***	288.5424***	3.8733***	59.186***	76.0951***	**5.925*** **

Li test: critical values at 1% level= 2.33(∗∗∗); 5% level= 1.64(∗∗); 10% level= 1.28(∗).

Table 4
Spearman rank correlations between all biased and non-normalized capacity notions.

Variables	BPC_o^{LR}	BPC_i^{LR}	BPC_o^{SR}	BPC_i^{SR}	SRC	C^{tang1}	C^{tang2}	LRC
BPC_o^{LR}	**0.855**[a]	*.571*[a]	*.652*[a]	*.560*[a]	*−.737*[a]	*−.629*[a]	*−.101*[b]	*−.753*[a]
BPC_i^{LR}	.672[a]	**0.998**[a]	*.294*[a]	*.991*[a]	*−.695*[a]	*−.707*[a]	*−.388*[a]	*−.708*[a]
BPC_o^{SR}	.734[a]	.257[a]	**0.918**[a]	*.304*[a]	*−.631*[a]	*−.543*[a]	*−.106*[b]	*−.643*[a]
BPC_i^{SR}	.662[a]	.988[a]	.271[a]	**0.996**[a]	*−.694*[a]	*−.707*[a]	*−.404*[a]	*−.701*[a]
SRC	−.883[a]	−.713[a]	−.498[a]	−.708[a]	**0.967**[a]	*.939*[a]	*.469*[a]	*.975*[a]
C^{tang1}	−.851[a]	−.692[a]	−.543[a]	−.697[a]	.934[a]	**0.965**[a]	*.579*[a]	*.947*[a]
C^{tang2}	−.257[a]	−.353[a]	−.122[a]	−.379[a]	.479[a]	.641[a]	**0.981**[a]	*.462*[a]
LRC	−.944[a]	−.691[a]	−.646[a]	−.684[a]	.960[a]	.950[a]	.460[a]	**0.988**[a]

[a] Correlation is significant at the 0.01 level (2-tailed).
[b] Correlation is significant at the 0.05 level (2-tailed).

three conclusions emerge from studying Table 3. First, for the convex capacity notions (below the diagonal) all capacity concepts follow two by two significantly different distributions, though the Li-test statistics between BPC_i^{SR} and BPC_i^{LR} is only marginally significant. Second, for the nonconvex capacity notions (above the diagonal) almost all capacity concepts follow two by two significantly different distributions, though again the Li-test statistics between BPC_i^{SR} and BPC_i^{LR} is only marginally significant. One exception are SRC and LRC that have indistinguishable distributions. Third, all capacity notions follow different distributions under convexity compared to nonconvexity (on the diagonal), though the Li-test statistic is only marginally significant for BPC_i^{SR} and BPC_i^{LR} at the 10% level.

Table 4 reports the Spearman rank correlation coefficients for biased and non-normalized capacity notions. This table is structured in a similar way as Table 3. In this table, components on the diagonal (in bold) depict the rank correlation between the convex and nonconvex cases. The components under the diagonal show the rank correlation between convex capacities and the components above the diagonal show the rank correlation between nonconvex capacities.

The following three conclusions emerge from studying Table 4. First, for the convex results, one can observe that SRC and LRC have the highest rank correlation among cost-based capacity notions, and that BPC_i^{SR} rank correlates better with all cost-based capacity notions in absolute values than BPC_o^{SR}. By contrast, BPC_o^{LR} rank correlates in absolute values better with three out of four cost-based capacity notions than BPC_i^{LR}. BPC_i^{SR} rank correlates better with BPC_i^{LR} than BPC_o^{SR} correlates with BPC_o^{LR}. Finally, the long-run plant capacity concepts rank correlate better among themselves than the corresponding short-run concepts.

Second, for the nonconvex results, exactly the same first two conclusions emerge. BPC_o^{LR} and BPC_i^{LR} both rank correlate in absolute values better with two out of four cost-based capacity notions. The input-oriented plant capacity concepts rank correlates better among themselves than the output-oriented plant capacity concepts. Finally, the long-run plant capacity concepts rank correlate better among themselves than the corresponding short-run concepts. Third, comparing convex and nonconvex results, the rank correlations are remarkably high overall among cost-based capacities notions, and these are highest for BPC_i^{SR} compared to BPC_o^{SR} and highest for BPC_i^{LR} compared to BPC_o^{LR}.

4.3. Comparing unbiased and normalized capacity utilization notions

Turning now to the unbiased and normalized capacity utilization notions, we develop a structure of arguments close to the one in the previous subsection. Table 5 lists descriptive statistics for all unbiased and normalized capacity utilization notions similar to Table 2. We again report the average, the standard deviation, and the minima and maxima depending on the context.

We first briefly comment on some of the convex averages to elucidate the underlying CU concepts. First, the average PCU_o^{SR} of 0.710459 means that the efficient outputs are situated at 71% of maximal efficient outputs. Second, the average PCU_i^{SR} of 1.733724 implies that variable inputs must be scaled up about 73% from where production is initiated to be able to produce the current level of outputs. The long run plant capacity concepts are very similar in interpretation. Third, the average SRC_N of 0.331119 means that the minimum of the short-run total cost function is at about 33% of observed costs. The other cost-based CU notions have very similar interpretations.

In this case, the relations between convex and nonconvex results are determined by the relations described in Proposition 3.2. First, ignoring the five CU notions that cannot be ranked, on average convex and nonconvex results are rather markedly different for the three other CU notions (i.e., SRC_N, C_N^{tang1} and LRC_N). Second, the range of the results differ sometimes. But, some share one of the extremes, except for PCU_i^{SR}, PCU_i^{LR} and C_N^{tang2} for which the range is again identical.

Table 6 reports the Li-test statistics and it is structured in a similar way as Table 3 above. A glance at Table 6 yields the following conclusions. First, for the convex capacity notions (below the diagonal) almost all capacity concepts follow two by two significantly different distributions, except PCU_o^{LR} and LRC_N that have indistinguishable distributions. Second, for the nonconvex capacity notions (above the diagonal) all capacity concepts follow two by two significantly different distributions. Third, all capacity notions follow different distributions under convexity compared to nonconvexity (on the diagonal).

Table 5
Descriptive statistics for all unbiased and normalized CU-notions.

Convex	PCU_o^{LR}	PCU_i^{LR}	PCU_o^{SR}	PCU_i^{SR}	SRC_N	C_N^{tang1}	C_N^{tang2}	LRC_N
Average	0.297313	1.776266	0.710459	1.733724	0.331119	0.43418	0.391276	0.260619
Stand. Dev.	0.196011	1.68314	0.221112	1.636011	0.173495	0.189827	2.280111	0.161353
Minimum	0.005251	1	0.070056	1	0.053327	0.103932	0.017937	0.036357
Maximum	1	21.14141	1	21.14141	1	1	45.5131	1
Nonconvex	PCU_o^{LR}	PCU_i^{LR}	PCU_o^{SR}	PCU_i^{SR}	SRC_N	C_N^{tang1}	C_N^{tang2}	LRC_N
Average	0.371884	2.584806	0.690958	2.539953	0.464158	0.629439	0.372487	0.378417
Stand. Dev.	0.252098	2.16346	0.244674	2.156585	0.253099	0.247589	2.270101	0.218591
Minimum	0.01639	1	0.096771	1	0.069012	0.133735	0.017937	0.039328
Maximum	1	21.14141	1	21.14141	1	1	45.5131	1

PCU_o^{LR}: Unbiased long-run output-oriented plant capacity utilization ($PCU_o^{LR}(x, y \mid VRS, .)$).
PCU_i^{LR}: Unbiased long-run input-oriented plant capacity utilization ($PCU_i^{LR}(x, y \mid VRS, .)$).
PCU_o^{SR}: Unbiased short-run output-oriented plant capacity utilization ($PCU_o^{SR}(x, x^f, y \mid VRS, .)$).
PCU_i^{SR}: Unbiased short-run input-oriented plant capacity utilization ($PCU_i^{SR}(x, x^f, y \mid VRS, .)$).
SRC_N: Normalized short-run total cost ($C(y, w^v, x^f \mid CRS, .)/wx$).
C_N^{tang1}: Normalized tangency cost with modified fixed inputs ($C^{tang1}(y, w, x^{f*} \mid VRS, .)/wx$).
C_N^{tang2}: Normalized tangency cost with modified outputs ($C^{tang2}(y(p, w^f, x^f), w, x^f \mid VRS, .)/wx$).
LRC_N: Normalized long-run total cost ($C(y, w \mid CRS, .)/wx$).

Table 6
Li-test between all unbiased and normalized CU-notions.

Variables	PCU_o^{LR}	PCU_i^{LR}	PCU_o^{SR}	PCU_i^{SR}	SRC_N	C_N^{tang1}	C_N^{tang2}	LRC_N
PCU_o^{LR}	**5.1188*** **	118.4083***	54.4991***	116.8833***	11.2028***	33.7377***	8.2625***	3.4488***
PCU_i^{LR}	142.2835***	**27.068*** **	94.1687***	−2.8477***	106.417***	92.5103***	144.72***	118.3195***
PCU_i^{SR}	57.2157***	68.3072***	**10.2515*** **	96.2052***	25.1713***	4.0829***	83.4402***	49.933***
PCU_o^{SR}	155.5302***	−2.9622***	173.8835***	**31.0053*** **	105.8964***	93.404***	143.2459***	116.4413***
SRC_N	5.9226***	124.2871***	288.6082***	174.2108***	**6.6207*** **	9.9527***	20.4263***	3.7252***
C_N^{tang1}	34.943***	106.7562***	133.9842***	174.24***	32.1426***	**25.0337*** **	58.8827***	27.8878***
C_N^{tang2}	3.3979***	142.0431***	295.9331***	174.2678***	66.8757***	67.0158***	**−2.8515*** **	14.5216***
LRC_N	0.7119	156.8096***	288.5418***	288.5424***	3.8733***	59.186***	76.0951***	**12.6322*** **

Li test: critical values at 1% level= 2.33(∗∗∗); 5% level= 1.64(∗∗); 10% level= 1.28(∗).

Table 7
Spearman rank correlations between all unbiased and normalized CU-notions.

Variables	PCU_o^{LR}	PCU_i^{LR}	PCU_o^{SR}	PCU_i^{SR}	SRC_N	C_N^{tang1}	C_N^{tang2}	LRC_N
PCU_o^{LR}	**0.797**[a]	.502[a]	.517[a]	.455[a]	.164[a]	−0.073	−.605[a]	.162[a]
PCU_i^{LR}	.571[a]	**0.899**[a]	.426[a]	.972[a]	.391[a]	.260[a]	−.689[a]	.484[a]
PCU_o^{SR}	.425[a]	.197[a]	**0.706**[a]	.418[a]	.253[a]	0.042	−.403[a]	.224[a]
PCU_i^{SR}	.565[a]	.985[a]	.181[a]	**0.888**[a]	.443[a]	.307[a]	−.661[a]	.521[a]
SRC_N	0.054	.617[a]	−.241[a]	.650[a]	**0.893**[a]	.734[a]	0.041	.935[a]
C_N^{tang1}	−.527[a]	.138[a]	−.295[a]	.156[a]	.569[a]	**0.807**[a]	.273[a]	.769[a]
C_N^{tang2}	−.876[a]	−.552[a]	−.326[a]	−.534[a]	−0.026	.630[a]	**0.997**[a]	0.016
LRC_N	0.073	.737[a]	−0.085	.749[a]	.894[a]	.585[a]	−0.074	**0.957**[a]

[a] Correlation is significant at the 0.01 level (2-tailed).

Table 7 reports the Spearman rank correlation coefficients for unbiased and normalized capacity utilization notions. As in Table 4, the components on the diagonal show the rank correlation between convex and nonconvex cases. The components under the diagonal show the rank correlation between convex CU notions, and the components above the diagonal show the rank correlation between nonconvex CU notions.

A close look at Table 7 leads to the following three conclusions. First, for the convex results, one can notice that PCU_i^{SR} rank correlates better with all cost-based CU notions in absolute values than PCU_o^{SR}, except for the C_N^{tang1} CU notion. PCU_o^{LR} and PCU_i^{LR} both rank correlate in absolute values better with two out of four cost-based CU notions. The input-oriented plant capacity concepts rank correlates better among themselves than the output-oriented plant capacity concepts. Finally, the long-run plant capacity concepts rank correlate better among themselves than the corresponding short-run concepts. Furthermore, SRC_N and LRC_N again obtain the highest rank correlation among cost-based CU notions. Finally, PCU_o^{SR} and PCU_o^{LR} essentially have a zero correlation with LRC_N.

Second, for the nonconvex results, PCU_i^{SR} and PCU_i^{LR} both rank correlate better with all cost-based CU notions in absolute values than their output-oriented counterparts. The input-oriented plant capacity concepts rank correlates better among themselves than the output-oriented plant capacity concepts. Finally, the long-run plant capacity concepts rank correlate better among themselves than the corresponding short-run concepts. In addition, PCU_o^{SR} and PCU_o^{LR} have now a close to zero correlation with C_N^{tang1}. Third, comparing convex and nonconvex results, the rank correlations are remarkably high overall among cost-based CU notions, and these are highest for PCU_i^{SR} compared to PCU_o^{SR} and highest for PCU_i^{LR} compared to PCU_o^{LR}.

5. Conclusions

This contribution has set itself two major goals. A first major goal has been to make a theoretically coherent input-oriented comparison between the introduced technical and economic capacity notions. As a point of comparison, also the output-oriented plant capacity notion has been included. A second major goal has been to make this coherent input-oriented comparison among capacity notions using both convex and nonconvex technologies to assess the impact of the convexity axiom. Theoretically, the investigation of this convexity hypothesis has led us to establish the cases where plant and economic capacity concepts can be ordered a priori (see Propositions 3.1 and 3.2).

The empirical results have shown the following key results. First, there appears quite some heterogeneity among the different technical and economic capacity notions in terms of descriptive statistics. Second, formal testing has revealed that in almost all cases technical and economic capacity notions follow different distributions. Thus, each of these concepts seems to capture a different part of economic reality. Furthermore, each and every capacity concept seems also to follow almost always a different distribution under convexity and nonconvexity. Thus, convexity matters from a distributional viewpoint. Third, the study of Spearman rank correlation coefficients reveals that almost uniformly the input-oriented plant capacity notion correlates better with the cost-based capacity notions than the output-oriented plant capacity notion under nonconvexity (less pronounced so under convexity). Furthermore, the rank correlations are overall high for convex and nonconvex results. Thus, convexity seems to matter less from a ranking point of view.

Therefore, two key conclusions emerge from this contribution. First, the recently introduced input-oriented plant capacity notions lend themselves overall more naturally to comparisons with cost-based capacity notions than the more traditional output-oriented plant capacity notions. Thus, while the short-run output-oriented plant capacity notion enjoys some popularity in empirical applications (see the literature review in Cesaroni et al., 2017), applied researchers should probably consider using the new input-oriented plant capacity notions that are more in line with the traditional cost-based capacity notions widespread in economics in terms of both the resulting distributions and rankings.

Second, convexity matters also for both technical and economic capacity notions. Therefore, it seems essential to further empirically explore potential differences between estimates based on convex and nonconvex technologies and cost functions in even greater detail (e.g., the impact on economies of scope, the effect on mergers and acquisitions, etc.). Thus, even though theoretically the impact of convexity has been known for some time, it is important to further explore the impact of convexity on key economic value relations in practice. The current evidence provided shows that this impact is nonnegligible when measuring capacity and that convexification may not be harmless.

As an agenda for future research, we can mention three issues. First, it would be good if our empirical results regarding both the comparison of input-oriented technical and economic capacity notions as well as the impact of the convexity axiom in this context would be corroborated in additional empirical work by other researchers. Second, while the input-oriented plant capacity notion compares well with cost-based capacity notions, one may wonder whether the traditional output-oriented plant capacity would fit much better with capacity notions based on the revenue function (see, e.g., Lindebo et al., 2007 or Segerson & Squires, 1995). This conjecture remains to be explored.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2019.01.014.

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